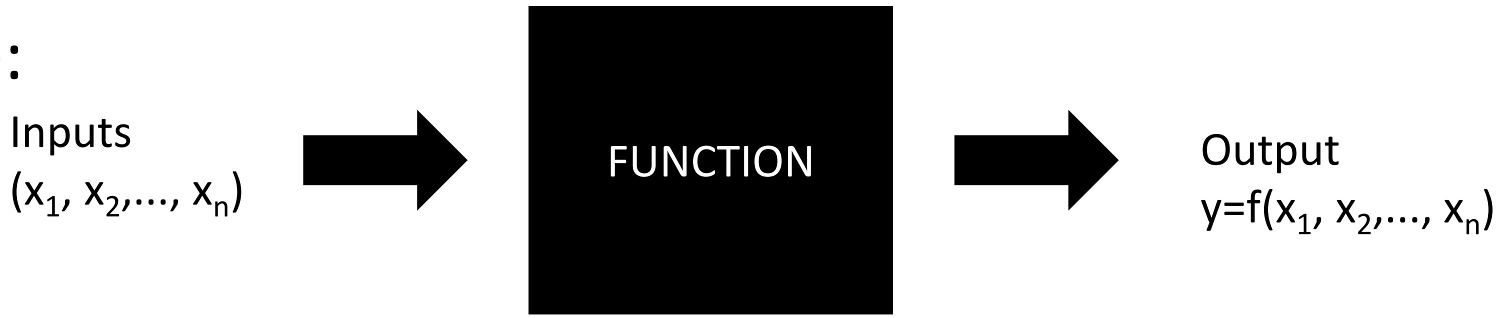
Math refresher for PPLE students

1. Functions

Functions

- You will meet functions in social sciences (yes, even in political sciences), and especially often in economics.
- Understanding what a function is fundamental.
- Function is a **relationship** between a number of factors that we can call inputs and a single factor that we can call output or the value of the function.
- For example:



Functions

- In the previous slide you have seen is a general formulation of a function, since we do not know or just do not want to specify the exact relationship. Then we just write:

$$y = f(x)$$

- That is, the variable y is a function of variable x. We call these variable, because we assume that they can change.
- If we know the fucntional realtionship then we can write for example:

$$y = 3x$$

- Which simply states that y can be obtained by multiplying x by three.
- This is also known as a **deterministic function**, since by knowing the values of x, one also knows the values of y perfectly.
- If we gain only imperfect knowledge of y then we have a random or stochastic function.

Functions

- We use functions to describe relationships. For example:
- If I earn 1 euro more, I usually consume 70 cents more.

$$consumption = a + 0.7 \cdot earning$$

- As you can see, there is still a parameter or coefficient here what I do not know based on the information I was given. Namely how much do I consume if I hav zero earning? If this is zero then a=0.

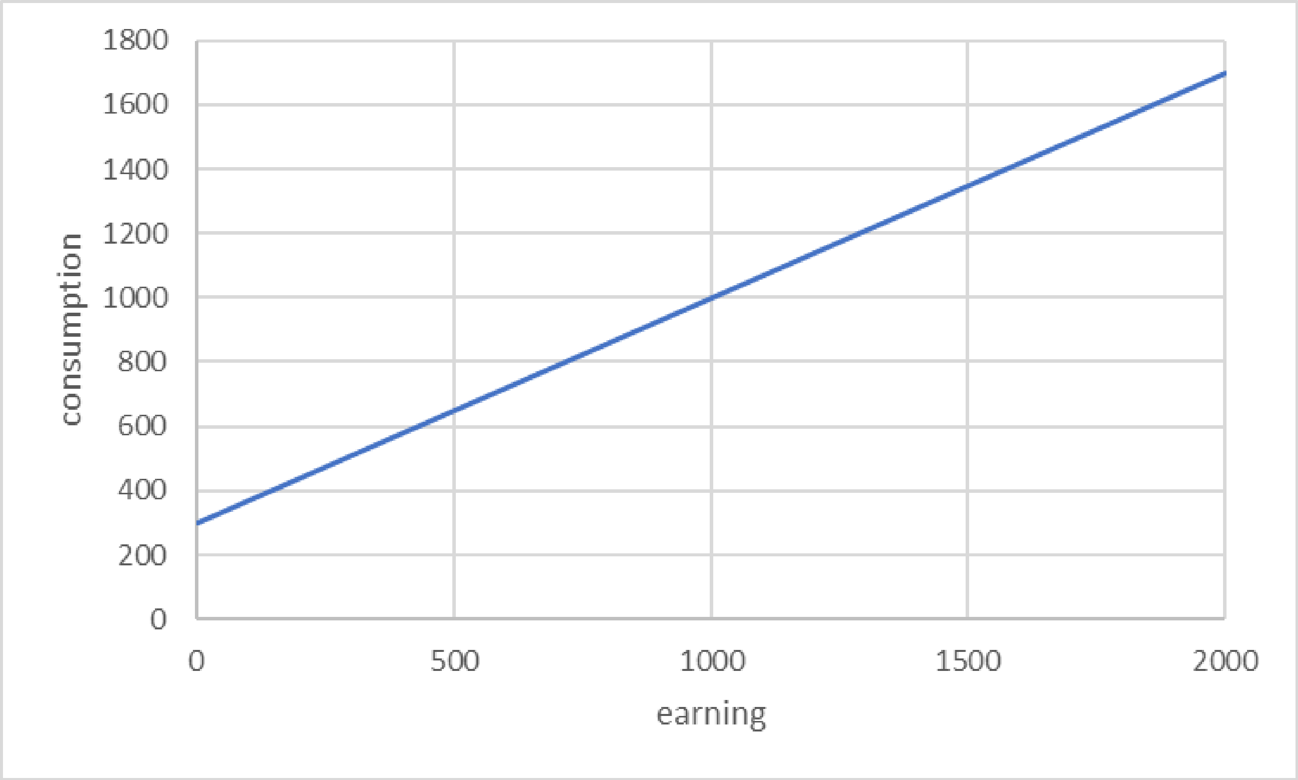
$$consumption = 0.7 \cdot earning$$

- May be this is not true: even if I have no earnings, my wife may buy stuffs for me or I can spend from my savings. Then even at zero earnings I will have a positive consumption, say, 300 euro. Then:

$$consumption = 300 + 0.7 \cdot earning$$

Linear Function

- The function we discussed is called linear function because the relationship between the explanatory variable (x or earning) and the dependent variable (y or consumption) can be visualized as a line.



$$y = a + b \cdot x$$

- Where a is the intercept or constant, which is the value of y if x=0
- And b is the coefficient of b or the **slope** of the line. This is the change in y if you increase x by one unit .

$$b = \frac{\Delta y}{\Delta x}$$

Finding the equation of the line

- What if I am given only two points which the line goes through? Could I guess the whole relationship?
- Let us assume that we observe the earnings and consumption expenditure of someone. We know that when she earned 2000 euro, she consumed 1800 euro, but when she earned only 1400 euro she consumed only 1200 euro. What is her linear consumption function?
- We have already seen that all we need is to find a and b.
- Since b is change in y over change in x: $b = \frac{\Delta y}{\Delta x} = \frac{1700 - 1200}{2000 - 1400} = 1$
- The intercept can be simply calculated either as:

$1800 = a + 2000 \rightarrow a = -200$ or $1200 = a + 1400 \rightarrow a = -200$

Finding the equation of the line

- So the consumption function is:

$$consumption = -200 + earning$$

- Graphically:

- Sometimes a negative intercept has no practical meaning. For example, negative consumption makes no sense, since human beings need basic necessities to survive.

Other functional forms

- What if consumption does not linearly depend on earnings? For example, a poor person may spend 90% of her earnings on consumption while a rich individual may only spend 50% of her earnings on consumer goods? What if the unit cost of producin cars decreases until a certain production level, but start to increase again beyond that?
- For such cases we use non-linear functions. The most populars are the following:

Quadratic function, power function, logarithmic function and the exponential function

The quadratic function

- Consider a car factory. If we increase production, the per unit cost (also known as average cost) of a car will decrease, since I can use the same amount of people and the same assembly line to produce more cars than before. Yet, if we above a certain output, we need to employ more and more people, and we need to repair our tools more often to keep up the production. Average costs start to increase.
- Such cases are often modelled as quadratic functions.

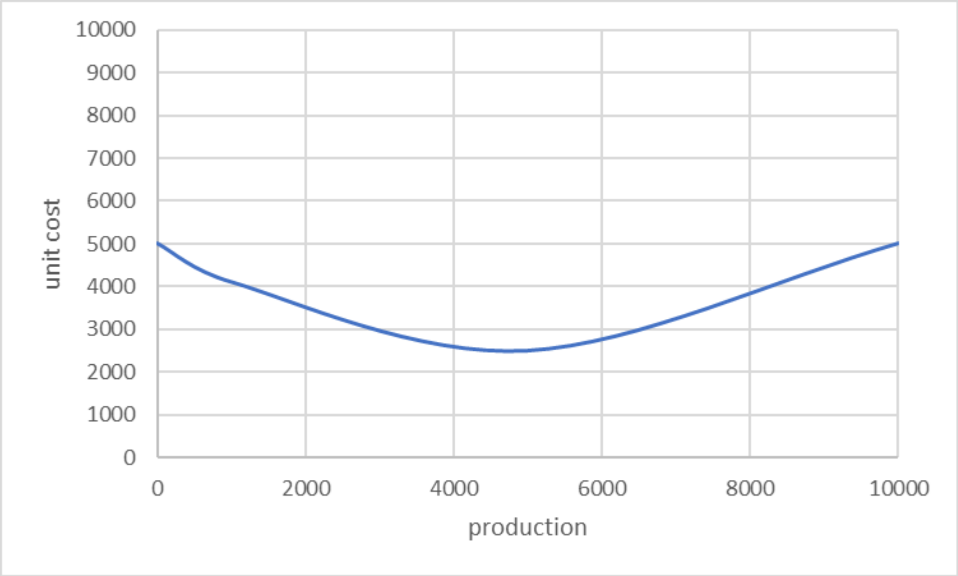
$$y = a + bx + cx^2$$

- The coefficent a is the value of y at x=0, but b and c has a different meaning than in the linear function.

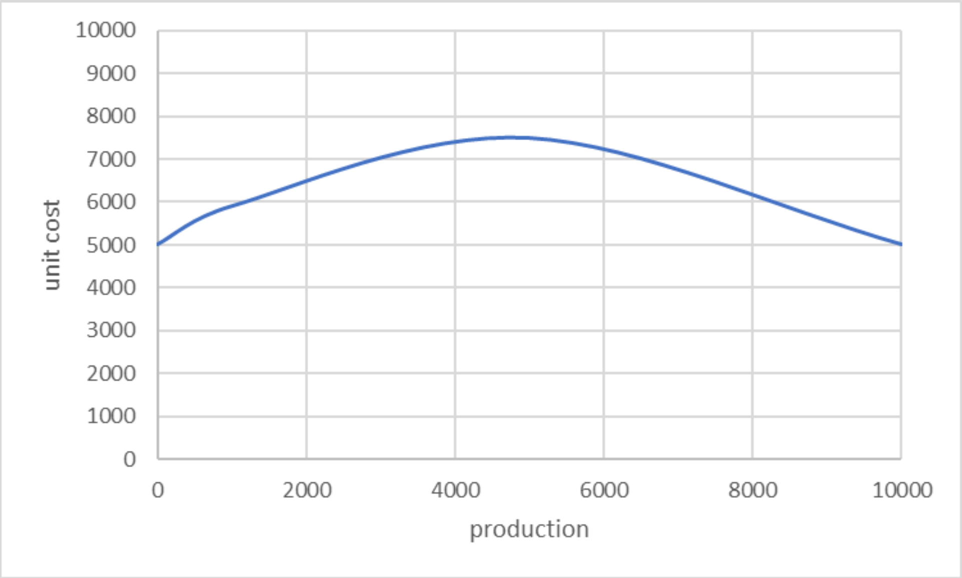
The quadratic function

- If b is negative and c is positive then we are going to get a U-shape relationship, if b is positive and c is negative, we will have a hump (inverse U-shape). If b and c are of the opposite sign then there is either a maximum of a minimum of y at a certain value of x.

$$y = 5000 - x + 0.0001x^2$$



$$y = 5000 + x - 0.0001x^2$$



Power function

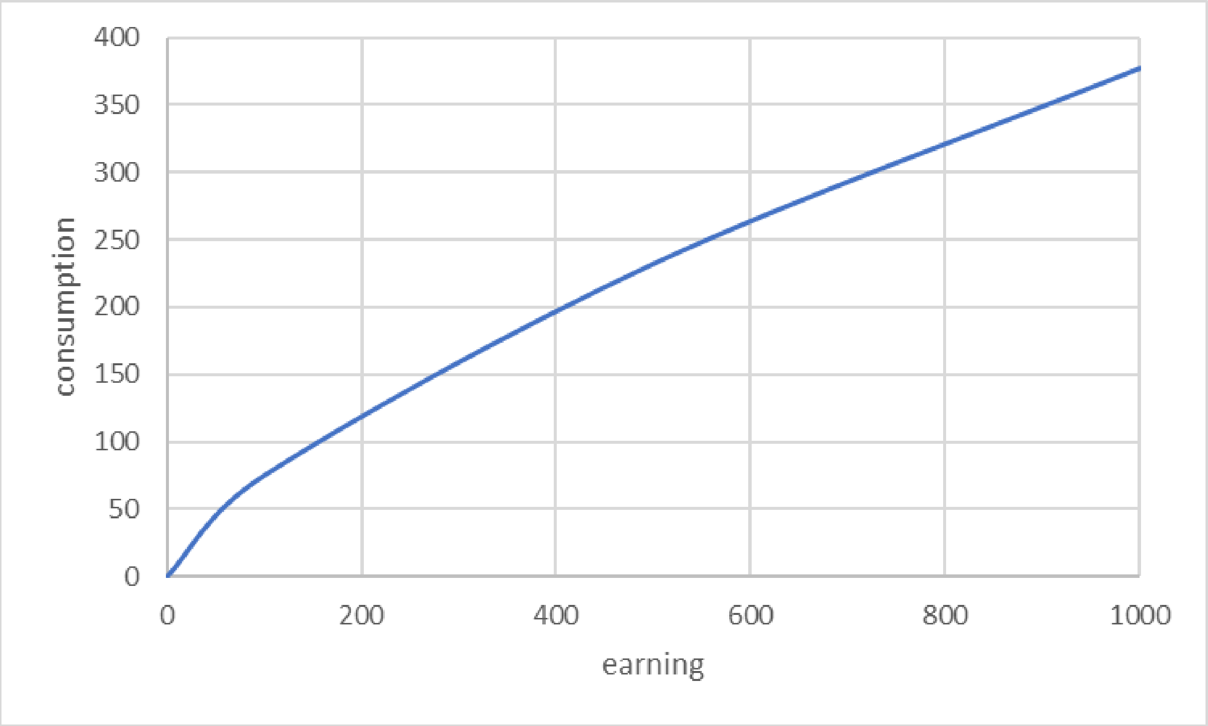
- Let us returns to the earnings/consumption problem. What if richer persons spend in relative terms less of their earnings on consumption than the poor? A useful functional form to desribe this is the **power function**.

$$y = ax^b$$

- When $0<b<1$, then the effect of x on y is decreasing as x grows, but if $b>1$ then x wil have a, increasing effect on y and the process is called explosive (i.e. grows very quickly).

$$y = 3x^{0.7}$$

$$y = 3x^{1.2}$$



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consumption
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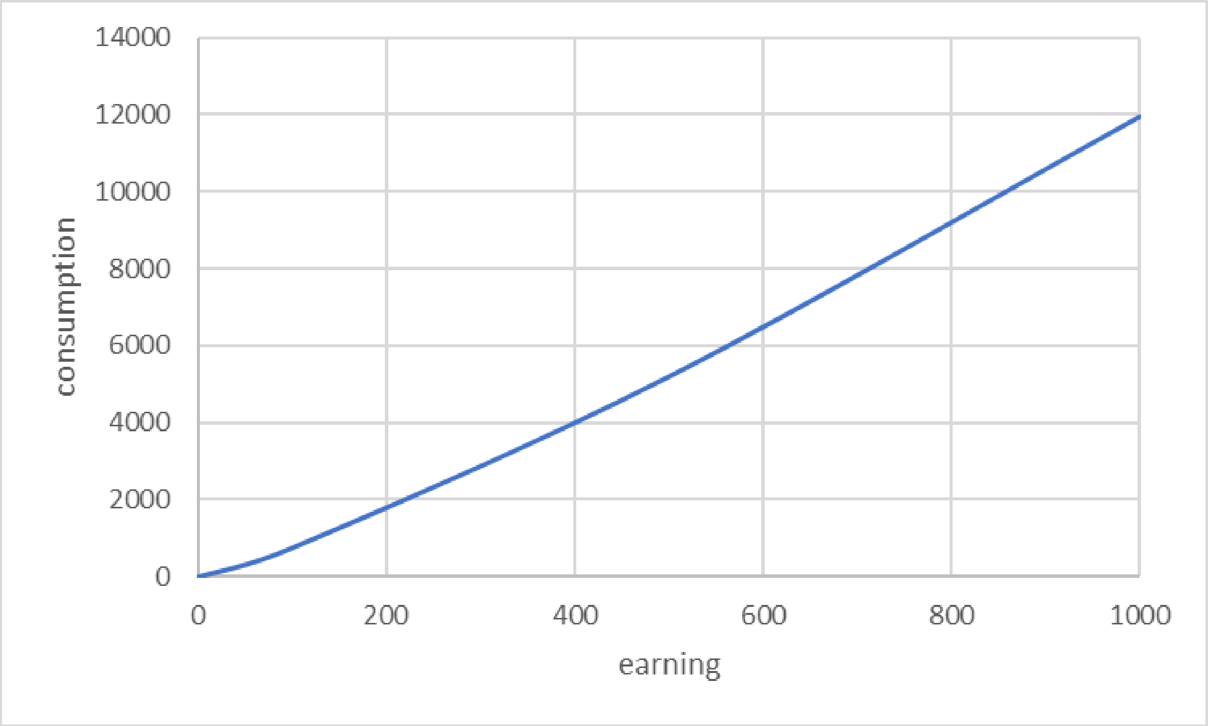
earning



14000
12000
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8000
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consumption
0
200
400
600
800
1000
earning

Power function

- Thsi function is also known as the constant elasticity function. Elasticity is an important concept you will learn about during the course.

$$y = ax^b$$

- Practically in the above power function, if x grows by 1% then y will grow by b %.

$$y = 3x^{0.7}$$

- For every % growth of x, y grows by 0.7%.

$$y = 3x^{1.2}$$

- For every % growth of x, y grows by 1.2% (explosive process).

Power function

- We may have even more than one explanatory variables. For example:

$$y = ax_1^{b_1} x_2^{b_2} \ldots x_k^{b_k}$$

- But do not worry about this: this is still a power function. The power function you will probably most often meet is the Cobb-Douglas-type production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \text{ with } 0 < \alpha < 1$$

- Where Y is the total output, K is the stock of capital (machines, equipment, building, inventories) and L is the total labor used.

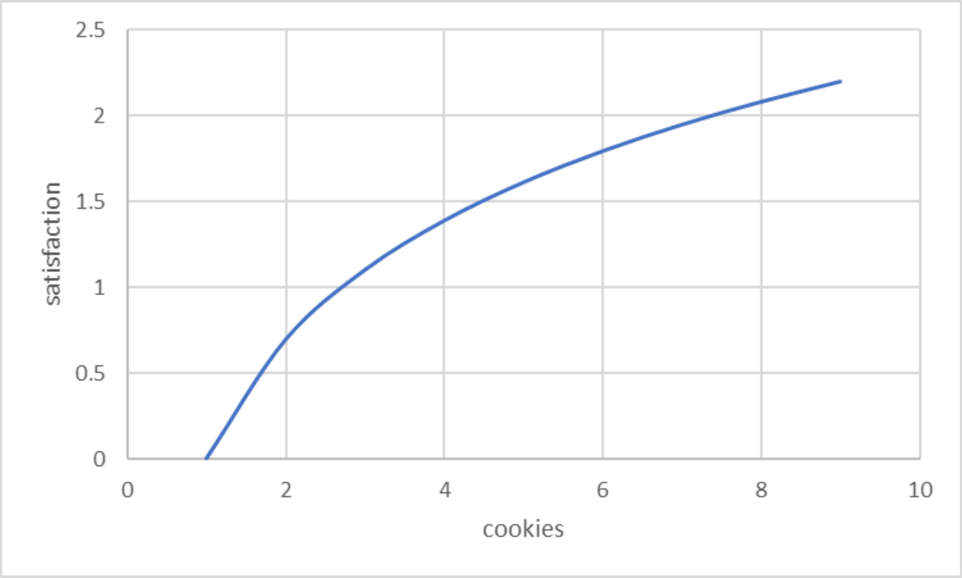
The (natural) logartihmic function

- The logartihmic function is used when we assume that an increase in x (input) does have a diminishing impact on the output variable y.
- For example, it is logical to assume that as I consume more and more of a product (say cookies), its effect on my degree of satisfaction (we call this satisfaction utility) will be lower and lower. This is the law of diminishing utility.
- The first cookie tastes the best, the second I may still like but possibly I would not enjoy the 20th cookie much.
- In such cases the logartihmic function is very useful.

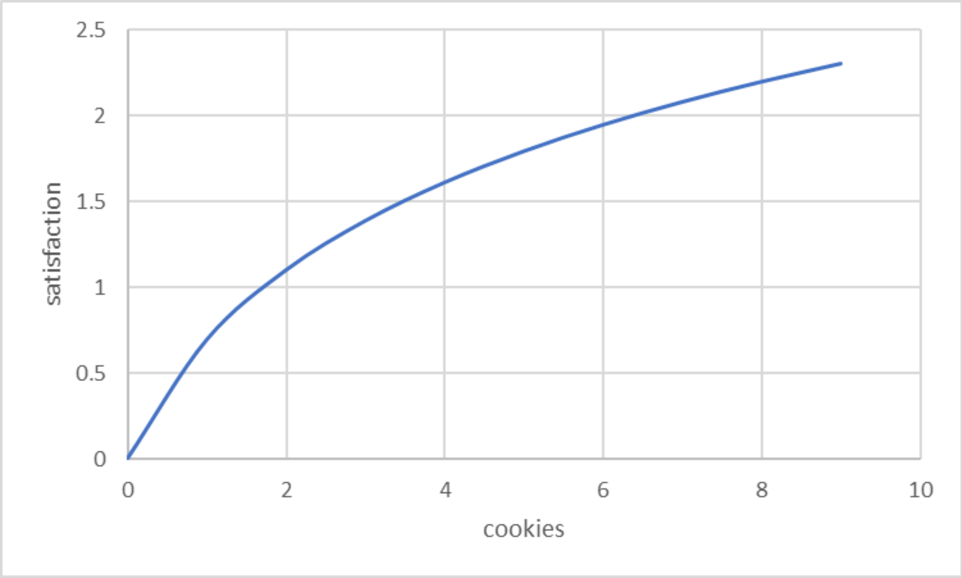
The (natural) logarithmic function

- Observe that at 0 the ln function is not defined.
- If you feel uneasy about this, just add 1 to x:

$$y = \ln(x)$$

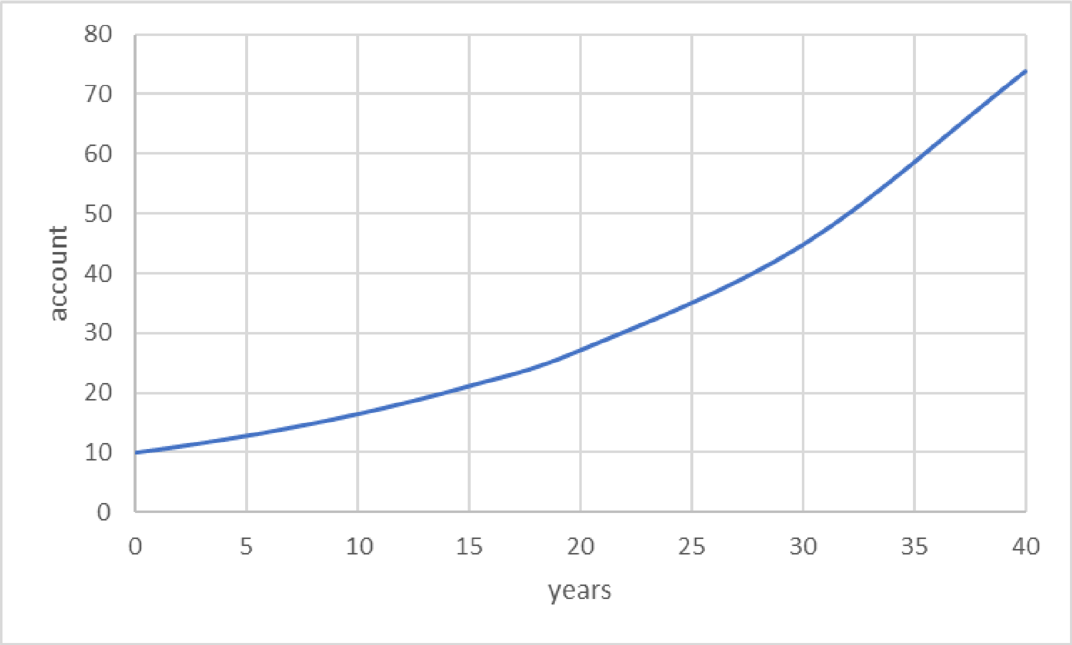


$$y = \ln(x+1)$$



The exponential function

- Exponential functions, such as $y = ae^{bx}$ are used to model quantities that increase or decrease by a constant percentage (e.g. interest, returns)
- Let, for example, *x* be the number of years, *b* the interest rate (say 5%), and *a* my initial balance (say 10 euro). Then, with compound interest, we can express my balance for any future period as follows:



$$y = 10e^{0.05x}$$

After 40 years, my initial 10 euro balance will grow in excess of 70 euro.