Math refresher for PPLE students

2. System of equations

What is a system of equations

- A system of equation is a collection of equations with the same unknowns.
- In economics you will encounter the simplest versions, hence we only take simple examples.
- Demand (D) and supply (S) as function of price (P)

$$D = 25 - 2P$$

$$S = -5 + P$$

- We know the demand and the supply function. This is not solvable yet, since we have two equations but three unknowns. We need the equilibrium assumption. D=S=Q, where Q is the equilibrium quantity.

System of equations

- Under the equilibrium (or market clearing) assumption the system becomes:

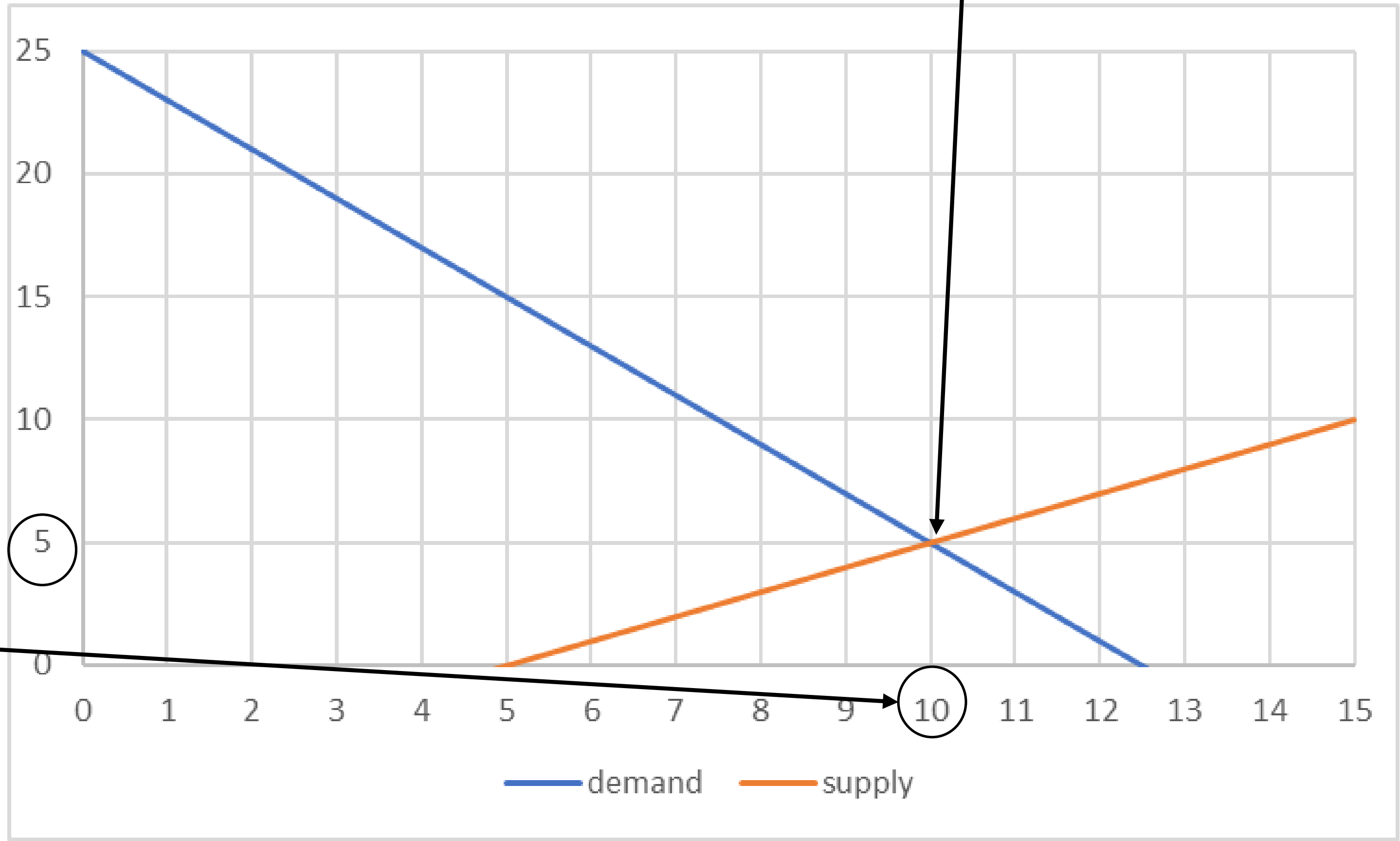
$$Q = 25 - 2P$$
$$Q = -5 + P$$

- Where hence we have two equations with two unknowns. This can be solved in most cases.
- You cannot solve it when one equation product of another equation by a constant. Then the two equations are linearly dependent and does not give extra information. You basically have two unknowns, one equation. The below equation, for example, cannot be solved:

$$-5Q = 25 - 5P$$
$$Q = -5 + P$$

Graphical solution

This is the equilibrium



These are the solutions.

Solution by substitution

- The idea is that I express any of the unknowns as the function of the other unknown and substitute it back into the other equation. For example:

$Q = -5 + P$ can be directly substituted into $Q = 25 - 2P$

- Which yields:

$$-5 + P = 25 - 2P$$

$$3P = 30$$

$$P = 10$$

Now, that we know the price in equilibirum, we can use that in any of the original equations:

$$Q = -5 + P = 5$$

Solution by elimination

- This is very useful to remember if you ever learn more math in the future.
- Remember that an equation remains true if I multiply both sides by the same number or if I add the same number to both sides.
- From the above rule it follows that in a system of equations you can also add up the respective sides of any two equations. Why? Because the two sides of an equation are equal. So you basically add the same number to both sides.

Solution by elimination

- Our equation is already ready for elimination:

$$Q = 25 - 2P$$

$$Q = -5 + P$$

- Let us subtract the second equation from the first:

$$Q - Q = 25 - 2P - (-5 + P)$$

$$0 = 30 - 3P \rightarrow P = 10$$

- Alternatively, we can multiply the second eqation by two, which makes our system look:

$$Q = 25 - 2P$$

$$2Q = -10 + 2P$$

Solution by elimination

- And now we can add the two equations up:

$$Q = 25 - 2P$$

$$2Q + Q = -10 + 2P + 25 - 2P$$

$$3Q = 15 \rightarrow Q = 10$$

- The idea is simple: you transform one equation so that by adding it to another equation at least one (but not all) unknown are eliminated from the problem.

An almost practical example

- A car producer produces two types: one is luxurious another is standard. The luxury car costs 24h to produce, the standard only 18h. We know that the total number of workhours available per day is 7200h. We know that the capacity of the plant is 360 cars per day. How many of the two types each should be produced so that we use up all our labor?
- We know that we have to use up all our hours:

$$7200 = 24L + 18S$$

- And we know that we can only produce 360 cars in total:

$$360 = L + S$$

An almost practical example

- Let us solve the system of equations by elimination:

$$7200 = 24L + 18S$$

$$360 = L + S \qquad /\text{x}24$$

- Now we have: $$7200 = 24L + 18S$$

$$8640 = 24L + 24S$$

- By subtracting the first from the second eqation we have:

$$1440 = 6S \rightarrow S = 240$$

- And then we can use substitution: $360 = L + 240 \rightarrow \text{L}{=}120$