Advanced time-series analysis (University of Lund, Economic History Department)

30 Jan-3 February and 26-30 March 2012

Lecture 5 Econometric techniques for stationary series 2: Distributed lag models, ARX type models, Koyck-transformation, Partial adjustment model, Granger causality.

5.a. Distributed lag models:

By distributed lag models we mean a dynamic specification where several lags of the explanatory variables are included.

$y_t = \beta_0 + \sum_{i=0}^{\infty} \beta_{i+1} x_{t-i} + u_t$, which is an infinite distributed lag model.

$y_t = \beta_0 + \sum_{i=0}^{k} \beta_{i+1} x_{t-i} + u_t$, which is a finite distributed lag model.

$y_t = \beta_0 + \sum_{j=1}^{p} \gamma_j y_{t-j} + \sum_{i=0}^{k} \beta_{i+1} x_{t-i} + u_t$, is an ARDL(p,k) type model

You already know the dynamic multipliers and the long-run effects. Just to refresh you memory:

$\frac{\partial y_t}{\partial x_{t-i}} = \beta_{i+1}$ for the two first DL models.

For the ARDL model you will need to specify p. If p=1, then:

$$(1-\gamma_1 L) y_t = \beta_0 + \sum_{i=0}^{k} \beta_{i+1} x_{t-i} + u_t \rightarrow$$

$\frac{\partial y_t}{\partial x_{t-i}} = \left(\beta_{i+1} + \gamma_1 \frac{\partial y_t}{\partial x_{t-i+1}} \right) = \left(\beta_{i+1} + \gamma_1 \left(\beta_i + \frac{\partial y_t}{\partial x_{t-i+2}} \right) \right) = \ldots = \sum_{j=0}^{i} \beta_{i+1-j} \gamma_1^j$ which will tend to zero in case of stationary process.

The long-run effects are much simpler to acquire:

$\sum_{i=0}^{\infty} \beta_{i+1}$ for the infinite DL, which requires for stationarity that $\lim_{i \to \infty} \beta_{i+1} = 0$, that is we believe that after a while the effect of a shock in x will become negligible.

$\sum_{i=0}^{k} \beta_{i+1}$ for the finite DL.

$\frac{\sum_{i=0}^{k} \beta_{i+1}}{1 - \sum_{j=1}^{p} \gamma_j}$ which requires the standard stationarity assumption for an AR model to be stationary.

Example of a finite DL

This exercise looks at the effect of inflation rate on treasury bill rates.

Dependent Variable: TBILRATE
Method: Least Squares
Date: 02/01/12 Time: 20:55
Sample: 1950Q1 2000Q4
Included observations: 204

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.306297	0.247077	13.38164	0.0000
INFL	0.490662	0.047659	10.29523	0.0000

R-squared	0.344138	Mean dependent var	5.229412
Adjusted R-squared	0.340891	S.D. dependent var	2.845157
S.E. of regression	2.309855	Akaike info criterion	4.522002
Sum squared resid	1077.757	Schwarz criterion	4.554533
Log likelihood	-459.2443	Hannan-Quinn criter.	4.535162
F-statistic	105.9917	Durbin-Watson stat	0.357704
Prob(F-statistic)	0.000000		

A static model suggests that treasury bill rates would not rise proportionally with inflation rate. This says basically that the real interest rate of treasury bills can reduce.

Dependent Variable: TBILRATE
Method: Least Squares
Date: 02/01/12 Time: 20:53
Sample (adjusted): 1951Q1 2000Q4
Included observations: 200 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.530400	0.239099	10.58308	0.0000
INFL	0.205888	0.061548	3.345159	0.0010
INFL(-1)	0.121411	0.063759	1.904221	0.0584
INFL(-2)	0.124488	0.062086	2.005099	0.0463
INFL(-3)	0.129683	0.063483	2.042800	0.0424
INFL(-4)	0.126496	0.060127	2.103815	0.0367

R-squared	0.516298	Mean dependent var	5.309650
Adjusted R-squared	0.503832	S.D. dependent var	2.815579
S.E. of regression	1.983272	Akaike info criterion	4.236914
Sum squared resid	763.0735	Schwarz criterion	4.335864
Log likelihood	-417.6914	Hannan-Quinn criter.	4.276957
F-statistic	41.41471	Durbin-Watson stat	0.165327
Prob(F-statistic)	0.000000		

With a finite DL we have a long-run effect around 0.71. This is more believeable.

An ARDL(1,1) model would lead to different estimate:

Dependent Variable: TBILRATE
Method: Least Squares
Date: 02/01/12 Time: 21:24
Sample (adjusted): 1950Q2 2000Q4
Included observations: 203 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.176677	0.102457	1.724403	0.0862
INFL	0.096173	0.019589	4.909429	0.0000
INFL(-1)	-0.037904	0.020477	-1.851019	0.0656
TBILRATE(-1)	0.926875	0.021868	42.38408	0.0000

R-squared	0.941005	Mean dependent var	5.249655
Adjusted R-squared	0.940116	S.D. dependent var	2.837425
S.E. of regression	0.694352	Akaike info criterion	2.127833
Sum squared resid	95.94286	Schwarz criterion	2.193118
Log likelihood	-211.9750	Hannan-Quinn criter.	2.154244
F-statistic	1058.063	Durbin-Watson stat	1.645657
Prob(F-statistic)	0.000000		

With the long-run effect being (.096-0.38)/(1-.927)=0.795, which is again higher.

In an error-correction representation you have exactly the same estimate:

Dependent Variable: D(TBILRATE)
Method: Least Squares
Date: 02/01/12 Time: 21:12
Sample (adjusted): 1950Q2 2000Q4
Included observations: 203 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.176677	0.102457	1.724403	0.0862
TBILRATE(-1)	-0.073125	0.021868	-3.343842	0.0010
INFL(-1)	0.058269	0.020122	2.895842	0.0042
D(INFL)	0.096173	0.019589	4.909429	0.0000

R-squared	0.129533	Mean dependent var	0.024187
Adjusted R-squared	0.116410	S.D. dependent var	0.738677
S.E. of regression	0.694352	Akaike info criterion	2.127833
Sum squared resid	95.94286	Schwarz criterion	2.193118
Log likelihood	-211.9750	Hannan-Quinn criter.	2.154244
F-statistic	9.870942	Durbin-Watson stat	1.645657
Prob(F-statistic)	0.000004		

Suggesting the long run relationship (cointegrating vector) being .058/0.073=0.795

This is because the error-correction representation is equivalent with an ARDL(1,1) type model. You can simply rewrite one into the other.

5.b. Some notable dynamic structural model types

<u>*Koyck-transformation:*</u>

You probably will not really need this, but this is classic, so you should hear about this at least once.

Let us assume you have an infinite DL model:

$y_t = \beta_0 + \sum_{i=0}^{\infty} \beta_{i+1} x_{t-i} + u_t$, where you assume that the effect of a shock in x will delay in geometric progression with time:

$\beta_{i+1} = \beta_1 \lambda^i$ that is, the effect of x_t on y_t is β_1, the effect of x_{t-1} on y_t is $\beta_1 \lambda$, the effect of x_{t-2} on y_t is $\beta_1 \lambda^2$ and so on, with $0 < \lambda < 1$.

Koyck arrived at the following transformation for this problem in 1954:

So the model can be rewritten as:

$$y_t = \beta_0 + \sum_{i=0}^{\infty} \beta_1 \lambda^i x_{t-i} + u_t = \beta_0 + \beta_1 \sum_{i=0}^{\infty} \lambda^i x_{t-i} + u_t$$

We use the quasi differencing trick as I showed you with the Cochrane-Orcutt process during the lecture:

$$\lambda y_{t-1} = \lambda \beta_0 + \beta_1 \sum_{i=0}^{\infty} \lambda^{i+1} x_{t-i-1} + \lambda u_{t-1}$$

So: $y_t - \lambda y_{t-1} = \beta_0 (1-\lambda) + \beta_1 x_t - \beta_1 \lambda^{\infty} x_{t-\infty} + u_t - \lambda u_{t-1} = \beta_0 (1-\lambda) + \beta_1 x_t + u_t - \lambda u_{t-1}$

which can be rewritten as:

$y_t = \beta_0 (1-\lambda) + \lambda y_{t-1} + \beta_1 x_t + u_t - \lambda u_{t-1}$, which is an ARMADL(1,1,0) model.

Forcing such a specification on our previous problem, we obtain:

Dependent Variable: TBILRATE
Method: Least Squares
Date: 02/01/12 Time: 21:50
Sample (adjusted): 1950Q2 2000Q4
Included observations: 203 after adjustments
Convergence achieved after 9 iterations
MA Backcast: 1950Q1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.356251	0.150377	2.369046	0.0188
INFL	0.068376	0.017924	3.814682	0.0002
TBILRATE(-1)	0.885001	0.027071	32.69202	0.0000
MA(1)	0.536167	0.062268	8.610646	0.0000

R-squared	0.946466	Mean dependent var	5.249655
Adjusted R-squared	0.945659	S.D. dependent var	2.837425
S.E. of regression	0.661439	Akaike info criterion	2.030710
Sum squared resid	87.06285	Schwarz criterion	2.095995
Log likelihood	-202.1171	Hannan-Quinn criter.	2.057122
F-statistic	1172.746	Durbin-Watson stat	2.266524
Prob(F-statistic)	0.000000		
Inverted MA Roots	-.54		

Which suggest a decay rate of 11 or 53% per quarter, depending on whether you trust the AR(1) or the MA(1) coefficient. Nevertheless, a Koyck-type model should really be used for problems when the assumption of geometric decay makes sense. If you would like to see a case when a Koyck model can be logical, let us take the relationship between consumption and income.
If agents do smooth their consumption over time, we can assume that past income effects will have a gradually decaying effect on present consumption. Let us estimate the ARMADL model:

Dependent Variable: REALCONS
Method: Least Squares
Date: 02/01/12 Time: 21:54
Sample (adjusted): 1950Q2 2000Q4
Included observations: 203 after adjustments
Convergence achieved after 12 iterations
MA Backcast: 1950Q1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-38.53771	7.731718	-4.984366	0.0000
REALCONS(-1)	0.754337	0.043486	17.34682	0.0000
REALGDP	0.174207	0.029778	5.850153	0.0000
MA(1)	0.256665	0.072342	3.547920	0.0005

R-squared	0.999820	Mean dependent var	3008.995
Adjusted R-squared	0.999818	S.D. dependent var	1456.900
S.E. of regression	19.67708	Akaike info criterion	8.816294
Sum squared resid	77050.34	Schwarz criterion	8.881579
Log likelihood	-890.8539	Hannan-Quinn criter.	8.842706
F-statistic	369054.2	Durbin-Watson stat	1.884159
Prob(F-statistic)	0.000000		
Inverted MA Roots	-.26		

Now the AR and the MA coefficients indicate the same lambda: 25% per quarter. The long run effect is 0.17/0.25=0.707. This is the long-run marginal propensity to save.

Partial adjustment model:

This is a simple way to create a dynamic model: you assume that there is an equilibrium value for y_t that depends on some exogenous variables:

$$y_t^* = \beta_0 + \sum_{i=1}^{k} \beta_i x_{k,t} + u_t$$

There is a tendency for y to return to this value, so any deviation from the equilibrium is eliminated:

$$y_t = y_{t-1} + \lambda(y_{t-1}^* - y_{t-1}) = (1-\lambda)y_{t-1} + \lambda y_{t-1}^*$$

from which:

$$y_t = (1-\lambda)y_{t-1} + \lambda\beta_0 + \lambda\sum_{i=1}^{k}\beta_i x_{k,t} + \lambda u_t$$

Which is again an ARX or ARDL model.

If we were to estimate the consumption-income relationship with a partial adjustment model we would obtain:

Dependent Variable: REALCONS
Method: Least Squares
Date: 02/01/12 Time: 22:09
Sample (adjusted): 1950Q2 2000Q4
Included observations: 203 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-27.39081	6.454799	-4.243479	0.0000
REALCONS(-1)	0.828396	0.036433	22.73760	0.0000
REALGDP	0.123504	0.024949	4.950190	0.0000

R-squared	0.999807	Mean dependent var	3008.995
Adjusted R-squared	0.999806	S.D. dependent var	1456.900
S.E. of regression	20.31709	Akaike info criterion	8.875470
Sum squared resid	82556.81	Schwarz criterion	8.924433
Log likelihood	-897.8602	Hannan-Quinn criter.	8.895278
F-statistic	519247.5	Durbin-Watson stat	1.432192
Prob(F-statistic)	0.000000		

Suggesting that the long-run equilibrium value of consumption is 0.1235/(1-0.828)=0.718 times the real GDP. And during every quarter 17,2% of the deviation from this equilibrium value is eliminated. The adjustment is rather slow.